

1. (8 points) Consider the function $f(x) = 2x^2 + 10x - 48$.

(a) Find the zeros of $f(x)$

$$0 = 2(x^2 + 5x - 24)$$

$$0 = 2(x + 8)(x - 3)$$

$$x = -8, 3$$

(b) Is $f(x)$ concave up, concave down, or neither? State why.

CCU

$$a = 2 > 0$$

2. (8 points) For each part of this problem, circle all of the expressions which could be formulas for the function described. There could be more than one answer for each part.

(a) The function $f(x)$ has a domain of all real numbers

1. $3x + 5$

2. $4x^2$

3. $\ln(x - 3)$

4. e^{3x}

5. $8x^5 - 4x^3 + 2x - 9$

6. $\sqrt{2x - 4}$

7. $\frac{3}{x - 5}$

8. None of these

1, 2, 4, 5

(b) The function $g(x)$ has a range of all real numbers

1. $3x + 5$

~~2. $4x^2$~~

3. $\ln(x - 3)$

4. e^{3x}

5. $8x^5 - 4x^3 + 2x - 9$

6. $\sqrt{2x - 4}$

~~7. $\frac{3}{x - 5}$~~

8. None of these

1, 3, 5

3. (6 points) The point $(3, -5)$ is on the graph of $f(x)$. What point must be on the graph of...

(a) $g(x) = f(x - 2) + 4$

$(5, -1)$

(b) $h(x) = f(x + 9) - 2$

$(-6, -7)$

4. (8 points) The tuition for Fred University is dependent on the number of credit hours a student is taking in a given semester. If a student is taking less than 12 credit hours, the student must pay \$800 per credit hour along with an additional fee of \$5000. If a student is taking 12 or more credit hours, they must pay 600 per credit hour with along with an additional fee of \$4000. The university will not allow any student to take more than 24 credit hours in a semester, and a student must take at least one credit hour.

(a) Find a piece-wise formula for the tuition, T , for a student taking C credit hours.

$$T = \begin{cases} 800C + 5000 & 1 \leq x < 12 \\ ~~6000~~ \\ 600C + 4000 & 12 \leq x \leq 24 \end{cases}$$

(b) What is the domain of this function?

$1 \leq x \leq 24$

5. (6 points) The profits of a new company are directly proportional to the cube root of the amount of money that it spends on advertising. The profit of the company will be \$300 if they spend \$1000 on advertising.

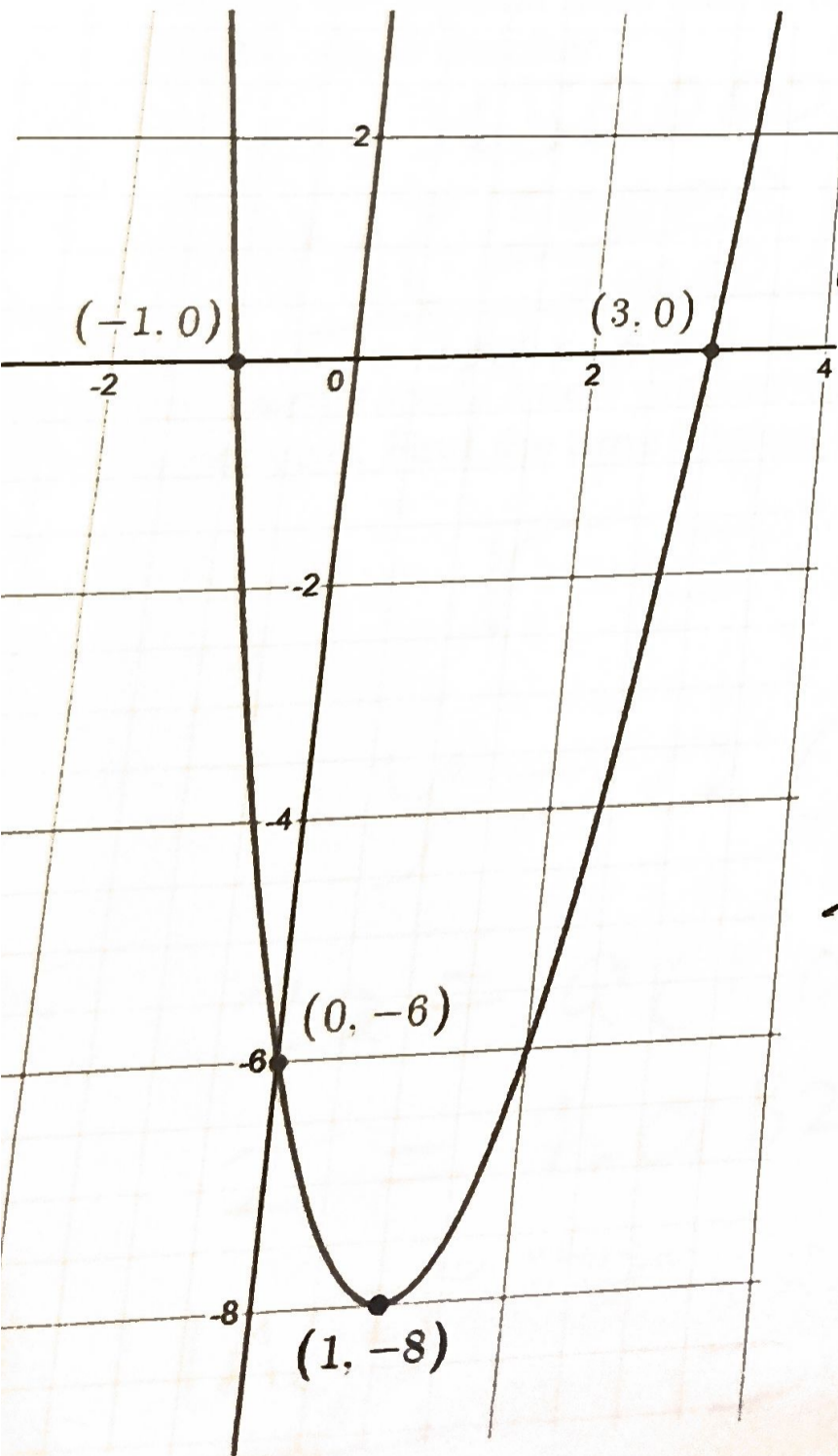
(a) Write a formula for the profit, P , as a function of the amount spent on advertising, A .

$$\begin{aligned} 300 &= k \sqrt[3]{1000} \\ 300 &= 10k \\ k &= 30 \end{aligned} \quad \Bigg| \quad P = 30 \cdot \sqrt[3]{A}$$

(b) Find the profit if \$125,000 is spent on advertising.

$$\begin{aligned} P &= 30 \cdot \sqrt[3]{125,000} = 30 \cdot 50 \\ &= 1500 \\ &\$1500 \end{aligned}$$

6. (6 points) Find the formula for the quadratic function $f(x)$ graphed below.



$$f(x) = a(x+1)(x-3)$$

$$-6 = a(0+1)(0-3)$$

$$-6 = -3a$$

$$a = 2$$

$$f(x) = 2(x+1)(x-3)$$

7. (9 points) The population of a town is 70,000 in the year 2025. Recall that a linear function has a general form of $P = mt + b$ and an exponential function has a general form of $P = a \cdot b^t$.

(a) If the population of the town is decreasing by 800 per year, find a formula for the function $P(t)$, the population t years after 2025.

$$P = -800t + 70000$$

(b) If the population is instead increasing by 3% per year, find a formula for the function $P(t)$, the population t years after 2025.

$$P(t) = 70000(1.03)^t$$

(c) Using your formula from part b, find the year the population will hit 150,000. Round to the nearest whole number.

$$150000 = 70000(1.03)^t$$

$$\frac{\ln\left(\frac{150000}{70000}\right)}{\ln(1.03)} = 25.784 \rightarrow 26$$

2051

(5 points) Dariya starts a new marketing firm. The profits of her business are increasing at a rate of 1.82% per year. Find the time it takes, in years, for her profits to double.

$$r = 1.82\% = 0.0182$$

$$y = a(1.0182)^t$$

$$2a = a(1.0182)^t$$

$$2 = 1.0182^t$$

$$\frac{\ln(2)}{\ln(1.0182)} = t \approx 38.43$$

years

9. (8 points) Determine if each function below is linear, exponential, or neither. Then, find a formula for the function or explain why it cannot be done.

(a)

x	1	3	5	7
$f(x)$	18	2	$\frac{2}{9}$	$\frac{2}{81}$

exp.

$$2 = ab^3$$

$$18 = ab^1$$

$$\Rightarrow \frac{1}{9} = b^2 \Rightarrow b = \frac{1}{3}$$

$$\text{so, } 18 = a\left(\frac{1}{3}\right) \Rightarrow a = 54$$

$$f(x) = 54 \cdot \left(\frac{1}{3}\right)^x$$

(b)

x	1	3	5	7
$f(x)$	4	12	-5	42

Neither. No constant slope or growth factor

(6 points) Jolena opens a bank account with an initial deposit of \$4,000. It earns interest at a nominal rate of 7% per year.

(a) Find the balance of their account after 6 years if interest is compounded as follows. Round each answer to the nearest dollar.

(i) Annually (once per year).

$$4000(1.07)^6 = \$6003$$

(ii) Daily (365 times per year).

$$4000\left(1 + \frac{0.07}{365}\right)^{365 \cdot 6}$$

$$= \$6088$$

(9 points) Let $f(x) = 8x - 2$, $g(x) = -4x + 3$ and $h(x) = e^x$. Find the following, and simplify your answers completely:

(a) $g(f(5))$

$$f(5) = 8(5) - 2 = 38$$

$$g(38) = -4(38) + 3 = -149$$

(b) $h(f(x))$

$$e^{8x-2}$$

(c) $f^{-1}(x)$

$$y = 8x - 2 \quad \rightarrow \quad y = \frac{x+2}{8}$$
$$x = 8y - 2 \quad \rightarrow \quad f^{-1}(x) = \frac{x+2}{8}$$
$$x+2 = 8y$$

2. (8 points) Consider the exponential function $Q = 10e^{-0.28t}$. Let t be measured in years.

7 (a) Determine if this function displays exponential growth or decay.

Circle one: **exponential growth** or **exponential decay**. Explain your answer in a sentence.

$$k = -0.28 < 0.$$

(b) Give the initial value for this function.

$$10$$

(c) Give the continuous growth rate for this function. Write your answer as a percentage.

$$-28\%$$

(d) Write the given function in the form $Q = ab^t$.

$$Q = 10(0.76)^t$$

(e) Find the percentage the function increases/decreases by per year. If the quantity is decreasing, write your percentage as a negative percentage.

$$-24\%$$

13. (10 points) Assume $R = f(t) = 1200(1.82)^t$ is the revenue, in thousands of dollars, of a business t years after 2025.

(a) Evaluate $f(6)$. Round to the nearest whole number.

$$1200(1.82)^6 = 43612$$

(b) Describe in words what the quantity you calculated in part (a) represents. Write your answer in a complete sentence with units.

Revenue in 2031 is \$43,612,000
or, 6 years after 2025, the revenue
is \$43,612,000

(c) Find a formula for $f^{-1}(R)$ in terms of R . Give an exact answer.

$$\frac{R}{1200} = 1.82^t \quad t = \frac{\ln\left(\frac{R}{1200}\right)}{\ln(1.82)}$$

(d) Evaluate $f^{-1}(20000)$. Round to the nearest whole number.

$$t = \frac{\ln\left(\frac{20000}{1200}\right)}{\ln(1.82)} = 4.698 \rightarrow 5$$

(e) Describe in words what the quantity you found in part (d) represents. Write your answer in a complete sentence with units.

5 years after 2025
the revenue is \$20,000,000

(3 points) Give an example of something that you learned in your course that will be useful in your future career and/or everyday life.